Internal Report no. 34

Procedure for calculating Electron Beam Dimensions in the HV-Terminal of the FEL

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Based on previously developed computer programs: ELOP and QUADOPT (a revised version), we summarize the procedure for calculating the e-beam dimensions at relevant locations along the transport line from the acceleration tube exit (screen $S_{\rm l}$) , through the wiggler, up to the deceleration tube entrance.

This report relies on previous reports # 2,3,10 in which the ELOP magnetic parameters were determined by comparison of simulation to experimentally measured data of the wiggler magnetic fields, and report # 29 in which "a standard case" set of beam parameters was used for simulation of transport along the entire terminal.

The report includes the following appendices:

Append. A: A mathcad program for preparing e-beam parameters.

Append. B: QUADOPT part I: Solution # 1 for optimal excitation currents of quads.

Append. C: QUADOPT part II: Solution # 2 for optimal excitation currents of quads.

Append. D: Tables of beam parameters and dimensions and quad excitation currents.

The procedure is based on a model of a Gaussian (or Eliptical) distribution of a finite emittance e-beam in $(x, y, \alpha_x, \alpha_y)$ phase space. Space charge effects are neglected.

The parameters of the ELOP model for the wiggler were found in previous reports (# 2,3,10), in which the ELOP simulation data was compared to the experimentally measured magnetic field along the wiggler. The quads parameters were determined from paraxial electron-optical analytical expressions, based on the manufacturer measured data of the magnetic gradient vs. excitation current and quad effective length (see Append. B).

Based on reports # 2-4,10 the measured magnetic field on axis is:

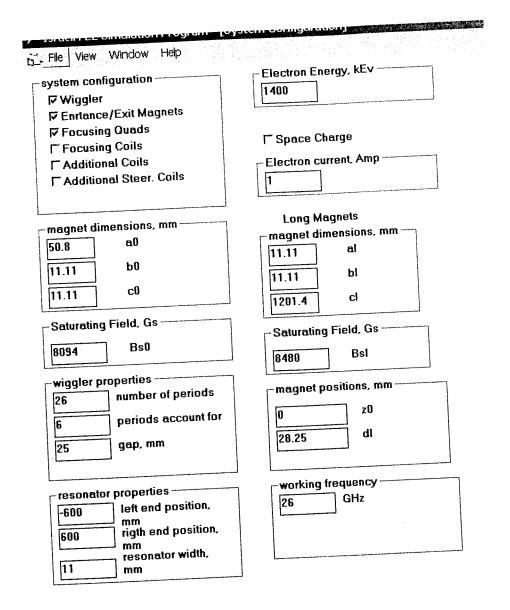
$$B_{y} = B_{w} \cos \left(\frac{2\pi}{\lambda_{w}} z\right) - \alpha_{R} x$$
$$B_{x} = -\alpha_{R} y$$

with

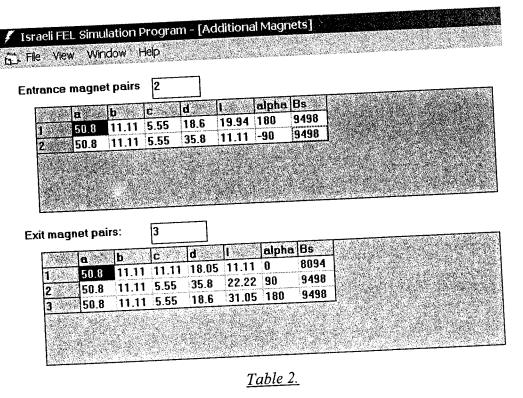
 $B_w = 1950 \pm 25 \,\mathrm{Gs}$

 $\alpha_R = 25 \pm 5 \,\mathrm{Gs/mm}$

Based on these reports it was found that the ELOP model for the wiggler magnets, longitudinal magnets and entrance/exit magnets, that best fit the Hall probe experimental data, is the one shown in Tables 1,2. In particular the best match for the saturation fields of the magnets was found to be:



<u>Table 1.</u>



 $B_{s0} = 8094 \text{ Gs}$ Wiggler: Long magnets: $B_{s0} = 8480 \text{ Gs}$

Drawing with ELOP the magnetic field on axis and on $X = \pm 1$ mm, $y = \pm 1$ mm, it was found with these input parameters that:

$$B_w = 1935 \text{ Gs} = 0.1935 \text{ T}$$

 $\alpha_R = 29.3 \text{ Gs/m} = 2.93 \text{ T/m}$

This is close enough to the experimental data. Therefore, we take the parameters of Tables 1,2 as the "Standard" parameters of the wiggler for the present experimental stage.

- 1. The difference between the saturation fields B_{s0} of the wiggler and longitudinal magnets is both because they were taken from different lots (purchased at different times), and because the longitudinal magnets are placed with some space between them, and B_{s0} represent an average value.
- 2. The B_{s0} values of the entrance/exit "correction" magnets were also determined by matching to the Hall probe experimental measurement of the field. However, since simulation indicated that the correction was not perfect, their parameter values in ELOP were modified to produce perfect correction of the electron trajectory for onaxis propagation all the way (in the experiment this will be done with the steering coils).

The procedure for computing the electron beam parameters (see also report#29) is as follows:

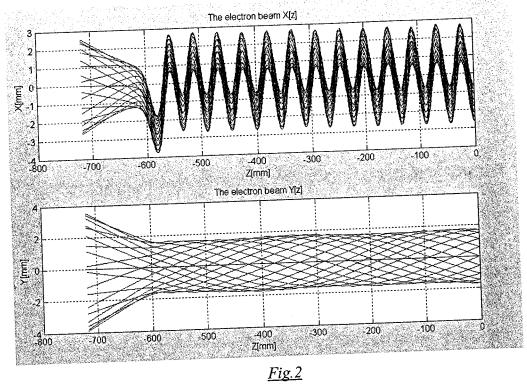
- 1. The beam parameters preparation program (Append. A) is run to determine: X_w, X_b,
- 2. The optimal center electron trajectory is found by running ELOP from starting point z = 0 to z = -700mm and z = +700 mm with initial conditions:

 $\alpha_{v}(0) = 0$ Y(0) = 0, $X(0) = -X_w, \quad \alpha_x(0) = 0,$

- 3. The values of X(0), and possibly the correction magnets parameters can be slightly changed until perfect on-axis propagation is obtained in and out of the wiggler.
- 4. ELOP is run from z = 0 to $z = \pm 700$ for a given emittance value with initial beam parameters X_b, Y_b calculated in step 1. These parameters can also be slightly adjusted until scallop-free beam propagation (in both x and y dimensions) is obtained inside the wiggler.
- 5. The beam is now propagated up to the screens positions z(S2) = -719mm, z(S3) =+ 813mm, and the optimal beam spot dimensions on the screens are determined. See
- 6. The virtual waist size and position of the beam entering the wiggler is found by starting the beam from the final position (z = -719mm) of the previous ELOP run and propagating it forward up to $z\sim$ -500mm while all the wiggler magnets are extinguished. The waist sizes Wox, Woy and positions Zwx, Zwy can be measured accurately after reading the data magnifying the drawing with Matlab or Mathematica. See Figs. 3,4.

Fig. 1

Beam diameter on S2. \mathcal{O}_x =5.2mm, \mathcal{O}_y =7.5mm. Start point Z=0, end point Z=-719.



Beam diameter on S3. \mathcal{O}_x =8.7mm, \mathcal{O}_y =11.2mm. Start point Z=0, end point Z=+813.

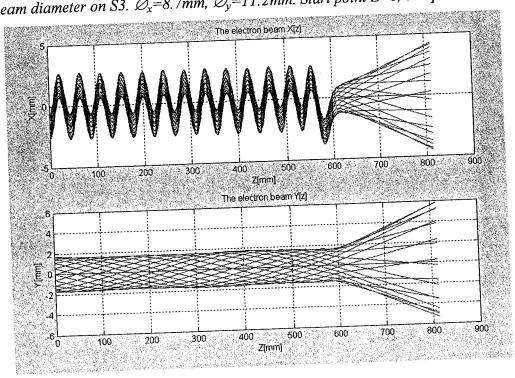
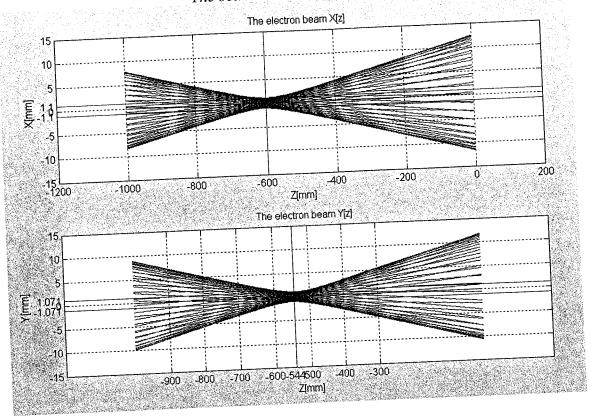
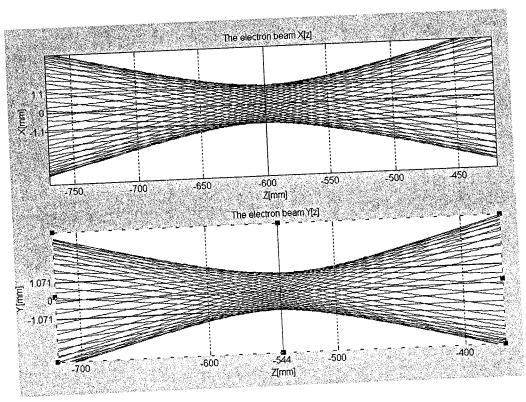


Fig.3
The beam virtual waist



Magnifying and determining the beam waist sizes $2W_{0x}$ =2.200mm, $2W_{0y}$ =2.142mm and waist positions Z_{wx} =-600mm, Z_{wy} =-544mm



7. QUADOPT part I (Append. B) is operated with the exit virtual waist parameters calculated in the last step: W_{02x} , Z_{wx} , W_{02y} , Z_{wy} , and the given waist dimensions W_{01x} , W_{01y} on screen S_1 at position $Z_s = -2681$ mm (assuming a waist is formed there). This results in the optimal values for the quad excitation currents:

$$I_1 = |I_1|, \qquad I_2 = -|I_2|, \qquad I_3 = |I_3|, \qquad I_4 = -|I_4|$$

In this solution and our current polarity definition, the positive current quads Q_1,Q_3 focus the beam in the x dimension and defocus it in the y dimension and vice versa for Q_2,Q_4 .

8. Due to symmetry of the quads positions relative to the wiggler, we can use the following choice for the quad excitation currents after the wiggler:

$$I_5 = -|I_4|, \qquad I_6 = |I_3|, \qquad I_7 = -|I_2|, \qquad I_8 = |I_1|$$

Alternatively the process of step 7 can be repeated with different choice of z_s , W_{01x} , W_{01y} for the exit waists near the deceleration tube entrance.

- 9. ELOP is run once again along the entire terminal length with the computed $I_1 I_8$, to verify scallop-free propagation in the wiggler and generation of waists (collimated beams) on both ends. See Fig. 5, and Table 3.
- 10. If desirable, one can calculate the second solution for the quad excitation currents using part II of QUADOPT (Append. C). In this solution $I_1 = -|I_1|$ (and so on) and Quad 1 first focuses the beam in the y dimension and defocuses it in the x dimension (and so on).
- 11. The beam dimensions at any desirable position z can be calculated by stopping the simulation at this point and measuring the beam widths. This can be more accurately done with Matlab or Mathematica.

Append. D shows the relevant dimensions for the "Standard case" example.

Warning notes

1. Because it so happens, that for the so defined "standard case", $k_{\beta x}$ is nearly equal $k_{\beta} (k_{\beta x} \approx k_{\beta})$, it turns out that $k_{\beta y} \approx 0$, which corresponds to very week focusing in the Y dimension. However, because $k_{\beta y}$ is obtained from subtraction of two big numbers, proportional to $k_{\beta 2}$ and $k_{\beta x}^2$, respectively any small deviation in the estimate of B_{s0} ($k_{\beta x} \propto B_{yy} \propto B_{s0}$) or B_{s1} ($k_{\beta x} \propto \sqrt{\alpha_R} \propto \sqrt{B_{sl}}$), will cause a big change in the value of Y_b ($\propto 1/\sqrt{k_{\beta y}}$).

Unfortunately the measurements of B_w (or B_{s0}) and α_R (or B_{s1}) are not accurate (in particular α_R). If fact, it seems from reports #4, 10 that an estimate of α_R =25Gs/mm (and correspondingly B_{s1} =7235Gs) would be more correct. Such an assumption will lead to quite different estimates of the beam parameters and particularly Y_b (and consequently the quad currents etc.). To avoid confusion, we stayed in this report with the "standard case" parameters that were used before also by Doron and Amir. However it will be desirable to check the deviations expected for different parameters, and eventually compare to the experimental results.

2. It is very important to measure the virtual waist parameters accurately. Otherwise the QuadOpt solution does not produce scallop-free beam trajectories. This also indicates, that experimental transport is very sensitive to the quads current.

<u>Fig. 5a,b</u> ELOP run. Start point Z=0 going back to Z=-2680, end point Z=2680.

 $\underline{Fig.5a}$: Solution #I

$$\begin{split} &I_{Q1}\!=\!1.60356,\,I_{Q2}\!=\!-1.19626,\,I_{Q3}\!=\!-1.31153,\,I_{Q4}\!=\!0.69281,\\ &I_{Q8}\!=\!1.60356,\,I_{Q7}\!=\!-1.19626,\,I_{Q6}\!=\!-1.31153,\,I_{Q5}\!=\!0.69281 \end{split}$$

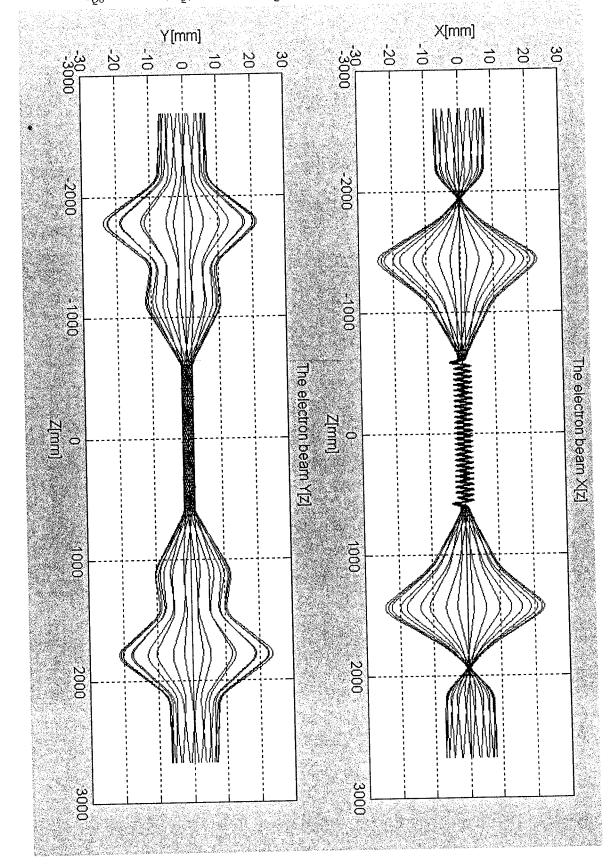


Fig.5b :Solution #II

$$\begin{split} I_{QI} = -1.67594, \ I_{Q2} = 1.22262, \ I_{Q3} = -1.28847, I_{Q4} = 0.65607, \\ I_{Q8} = -1.67594, \ I_{Q7} = 1.22262, \ I_{Q6} = -1.28847, I_{Q5} = 0.65607 \end{split}$$

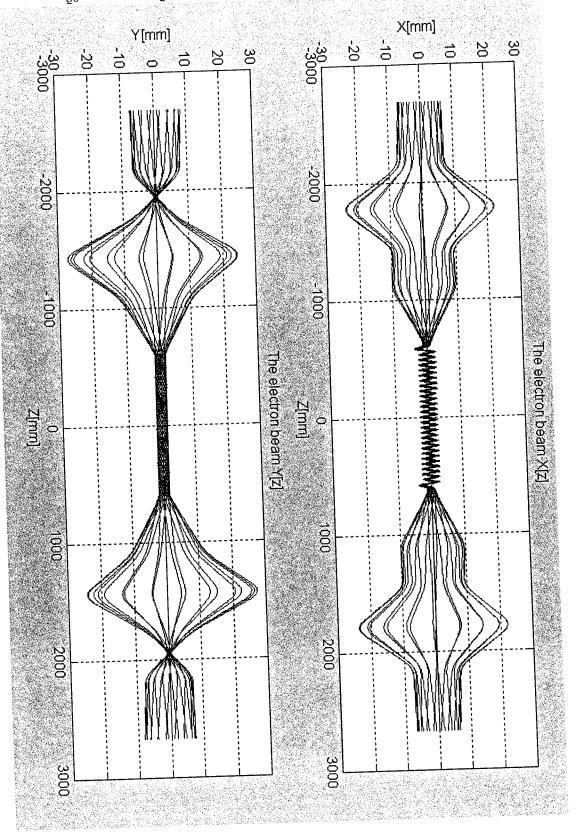


Table 3.

ELOP input window for e- beam propagation (R_{x0} , R_{y0} entrance beam radii of waist, θ_x , θ_y : entrance angular spread) These correspond to ε =22 π mm-mrad.

